

THEORY AND SYSTEM

I.

INTRODUCTION

Science was a *theory*. The term had entered the circuit of the accepted concepts as a matter of course, as a real Cartesian idea with the support in its own evidence.

Our times converted the theory – i.e. the science into a system. As it will be further seen, the consequences change essentially the notion of science.

Let us first explain intuitively the difference between «theory» and «system».

The first thinker who built up a theory of science, and established its structure, was Aristotle. The significance of the term «theory» should therefore be looked for in the language of the Greek philosophers.

The word «theory» – θεωρία – meant primarily «contemplation», «vision» and only later on it merged into ἐπιστήμη, science or knowledge.

The sense distinctions are more obvious if we analyze etymologically the Greek terms at issue.

Theory – θεωρία – comes from the verb θεωρία, to contemplate, to see directly; therefore, a corpus of immediate knowledge, directly acquired by intellectual intuition, has been originally called theory.

On the other hand, the term science – επιστήμη – stems from the verb – ιστήμη – which, among other things, stands for to establish, to order, made up of the particle επι, that (here) means «upwards». Therefore science signifies hierarchical establishment, by which the Greeks denoted for instance, geometry: a hierarchical ordering of truths (or of true propositions).

Such a hierarchical construction had to start from a group principles (theoretically obtained, i.e. directly), wherefrom, by way of proof, the other truths of theorems of the respective science were to be obtained.

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The name «system», appears even at the end of the Middle Ages but especially during the Renaissance and it will be assimilated also from the Greeks. The etymological sense of the term «system» is found in the Greek word συστημα (system), made up of the particle συν (with) and the verb σταω, to stand. In other words, συστημα means «to stand with the others», «to be put together with others», «to be tied together», «to be coordinated in a whole with other parts». Consequently, there is no longer a matter of construction, as for a science – επιστήμη – which starts from soem «thoretical» principles and comes down, by a series of hierarhical stages, to the truths of

science, but of juxtaposition of some propositions of truths; it is merely a matter of coherent internal structure of the corpus of propositions under consideration.

The idea of a system appears, as already mentioned, in certain Renaissance works, but it assumes a clear shape in Bartholomeus Keckermannus «*Systema Systematum*» – «The system of the systems» «posthumously published, Hanoviae, 1613); Clemens Timpler, *Logicae systema methodicum* – «The methodical system of logic» (Steinfurt, 1604); J.H. Alstedius, *Logicae systema harmonium*, «The harmonius system of logic» (Herbonae, 1612); Johannes Förster, *Systema problematum theologicorum*, «System of theological problems» (Wittebergae, 1610); a.s.o.

It therefore results that the distinction between science and system lies in the fact that science implies «theory» in the Greek meaning of the word, i.e. a series of directly obtained truths where they afterward hierarchically come from, while the system is exclusively founded on its coherence. *Science has its supporting foundation outside, whereas the system is exclusively justified by its internal construction.*

Of course, science also assumes an internal, coherent structure; but the existence of science is not exclusively due to this structure, that is in the turn an externally determined result. In other words, science implies the system but does not reduce to it, since this is only an aspect of the science, one of the spectral visions; the system is a necessary but not sufficient condition that a corpus of terms and propositions should build up a genuine science.

Briefly, science, in its etymological meaning, implies «theory» the direct knowledge of some elements or principles it ensues from; the system, since it does not assume the theory, is but a spectral photograph which shows only the anatomy of science and not its physiology too.

This conclusion, drawn only after considering the intuitive meaning of the notions at issue, will be rigorously developed in the following.

We shall point out that nowadays the idea of theory has been replaced by the idea of system and therefore the conception on science of our times bears a certain, quite particular significance, which we shall be emphasizing.

II.

ARISTOTLE'S CONCEPT OF THEORY

Here is how E.W. Beth summarizes the Aristotelian theory of science (1):
A deductive science is a system S of propositions that satisfies the following postulates.

- (1) Any propositions belonging to S should refer to a specific field of real entities.
- (2) If any proposition should belong to S, then any logical consequence of this proposition should belong to S.
- (3) If a certain proposition belongs to S, any logical consequence of this proposition should belong to S.
- (4) There is a (finite) number of terms in S, such that:
 - a) the significance of these terms is too obvious for additional explanations;
 - b) any other term in S is definable by means of those terms.
- (5) There is a (finite) number of propositions in S, such that:
 - a) the truth of these propositions is too obvious to need another proof;
 - b) the truth of any other proposition which belongs to S may be established by logical inference, starting from, those propositions.

The postulates (1), (2) and (3) are, respectively, called by Beth, *reality, truth* and the *deductivity* postulate. The postulates (4) and (5) together are the so-called *evidence postulates*; finally, the basic terms and propositions specified by the postulates (4) and (5) are called the principles of sciences (2).

We could *grosso modo* admit that the theoretical science, in Aristotle's conception, had that structure. But we shall point out that according to the Great Stagirite (3) «science deals with the universal and consists of necessary propositions».

What really strikes us first of all is that all those who discussed the Aristotelian conception in science disregarded the fact that this conception is above all «the science of the universal» – η σ'επιστημη του χαθδλου – and also what the scholastic logicians always repeated: *scientia est universalium* or even more *nulla est scientia fluxorum* («science belongs to the universals» or even more «there is no science of the ephemerals»). Beth, as generally all other contemporary logicians and mathematicians, who dealt with the Aristotelian conception of science, have neglected the fact that the Stagirite's science is the science of the universal and they confined themselves only to the analysis of the apodictic science, επιστημη αποδειχτιχη, exclusively conceived like the science of the proof.

But, if there is «no science except for the science of the universal», in order to clear the Aristotelian concept of science, one should start by firstly explaining the concept of universal – το χαθδλωσ – this has not been accomplished either by Beth or, before him, by F. Enriques (4) and I. Bochenski (5), a.s.o.

Yet the universal is perceived in the definition (that just for this reason cannot be but universal).

Without going more deeply into the notion of universal and of definition which expresses the essence – το τι εστι – of a thing, one cannot understand the theory of science as worked out by Aristotle.

But the Aristotelian universal is not the general, as usually understood in an inaccurate and somewhat naive way. Here is what Aristotle says on the universal: «we call universal that what is always and everywhere» αει και πανταχου (6).

This concept of «universal», characteristic of the Greek science, has been lost by the modern thought (with very few exceptions). The present science reduced the universal to the pure extensive notion of «mathematical set» or «class» and thus impoverished not only the philosophical but also the mathematical and logical thought.

It is obvious that by leaving the universal, i.e. the ontology, the essence-bearing being, modern thought has lost its whole Greek tradition. In this sense too, Hegel rightly said that «Aristotle's thesaurus is since centuries as well as unknown».

We shall refrain here from a further development of this topic; we however point out that the absence of the theory of universal when reproducing Aristotle's conception on science, cripples his theory and deprives it of what is central in the Stagirite's conception. This is our first objection, which will not halt us (we hope to tackle it again in another contribution) since we shall state a second one, referring this time not to an absence in the reproduction of Aristotle's thought, but to an unaccountable confusion in the interpretation of the texts.

This is what Beth reproaches to Aristotle's theory in the above quoted work (7): «the Aristotelian theory of sciences requires *metaphysics* as the science of principles». In other words, according to the same author, in order to account for the possible accepting of the principles for themselves, without proof, Aristotle needs the metaphysics, that is actually «*a research on foundations*» and that is the *first philosophy*. Beth thinks that Aristotle's theory of knowledge is analogous to the mystic doctrines (8) and the author concludes: «The Aristotelian theory of sciences has guided the scientific research until recently. This general acceptance of the Aristotelian theory of science implies the problems of his metaphysics and of his theory of knowledge» (9).

But Beth's assertion does not correspond to the texts. Here is how Aristotle describes science (10): «Any rational knowledge, be it learnt, or acquired, always has its roots in previous knowledge. Observation showed that this is valid for all sciences: indeed, this is the procedure of mathematics and without exception, of all arts». And the Stagirite goes on: «... the proved knowledge should result from premisses which are *true, first, immediate, better known and also known prior to conclusion*» (11).

Aristotle specifies that the proper object of science «is something that cannot be otherwise than it is» i.e. which exists necessarily – εξ αναγκαιον. But in order that not even the slightest doubt should exist about his intentions, which were beyond any metaphysical conception (in the description of science), Aristotle adds in the same paragraph: «If there is also another way of knowledge, this will be discussed later» (12).

Contrary to Beth's and other contemporary logicians' assertions, it results that the Stagirite does not apply in any way his metaphysics when describing the theory of science.

He establishes the *sine qua non* conditions of every theory, as such. We therefore can say, according to Aristotle's texts on the nature of the scientific knowledge (¹³): The object of science is the necessary and the necessary is acquired by proof; the principles where science begins should be better known previously and immediately (knowledge by way of proof is a mediate knowledge).

Consequently, for Aristotle, the scientific theory, assumes the following structure:

- 1) the principles of science, otherwise known than by proof, hence by immediate knowledge (¹⁴);
- 2) the theorems of science, known by proof, hence by mediate knowledge;
- 3) the method of the proof, which is especially the syllogism.

In this description of a scientific theory, Aristotle does not involve his metaphysical conception whatsoever. *This conception will be the solution of the problem: how is knowledge otherwise possible than by proof?*

He will prove, and we shall further exhibit his argumentation, that any theory should have this structure, since otherwise there will be no theory; he will still show the possibility to know the principles (i.e. otherwise than by proof).

We have consequently to distinguish between two things in Aristotle's conception on the knowledge. To mistake these two aspects of the Aristotelian theory of science means to mix what Aristotle himself, as already shown, had carefully separated.

III.

FORMAL SYSTEMS OF SCIENCE

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The Aristotelian conception on the science commanded the scientific research, being accepted as a matter of fact. To this effect, Beth writes (¹⁵): «Aristotle's theory of science has until quite recently directed research. Indeed, it was accepted with such a degree of unanimity, that nobody even thought of conferring special merit to Aristotle for his establishment of it or devoting special studies to its origins, its development and its further destiny».

Our times witnessed the frenzied struggle waged against the Stagirite's conception. Not against the conception of the structure of science – this is

not debatable – but against the idea that its principles should necessarily be known otherwise than the proved propositions.

Several developments led the scientists to mistrust «evidences». The possibility to construct non-euclidean geometries, by arbitrarily accepting Euclid's postulate or by not accepting it; the construction of some physico-mathematical theories – as for instance, the theory of relativity – by accepting certain postulates which oppose any intuition; the construction of polyvalent logics, where the principle of the excluded middle is no longer valid, a.s.o.; all these led to the conclusion that axioms, according to H. Poincaré are but «convenient conventions» (16).

In other words, since the axioms of science are conventions, they can bear neither the evidence character, nor can they assert themselves by some necessity outside science, the only obligation for their admittance being the fact that the corpus of notions and propositions of a science should «stand» consistently one close to the other. This is how a scientific theory merged into a «system».

The replacing of a «theory» by «system», for the above mentioned reasons, unleashed the war against evidence. Due either to the rational intuition or to the intellectual one, evidence has been wholly given up, in order that the system, so we are told, have the greatest possible objectivity. The renunciation to any possible content of the scientific language presumtively grants it the highest objectivity and this can be achieved only if signs, connected according to precise rules, which were given from the very beginning, are used. The system, as already explained in its etymological sense, had no connection either with any content be it sensitive or intellectual; it is a scheme, where certain contents may enter, but in itself it is merely an empty symbolic structure, a skeleton of signs.

The sign lacking any content is the basic element of the system. In this sense Hilbert will say: «*Also am Anfang, so heisst es hier, war das Zeichen*» (Therefore, at the beginning, as it ensues, there was the sign) (17).

The axiomatic method, whose results could not be underestimated, conjugated with formalism, led to the construction of formal systems which replace the theory of science according to the Aristotelian conception. A scientific theory is now a formal system and it will be so much the more «scientific» and «objective» as it will be more formalized.

But what is a formal system? Here is a description of the formal systems given by A. Fraenkel and J. Bar-Hillel. A formal system is determined by the following five sets (18):

- 1) The set of *primitive symbols*, that constitutes the *primitive vocabulary*, divided into *variables*, *constants* and *auxiliary symbols*.
- 2) The set of *terms*, as subset of the set of expressions determined by effective rules.
- 3) The set of *formulae*, as subset of the set of expressions determined by effective rules by means of the notion, *term*.
- 4) The set of axioms, as subset of the set of formulae.

- 5) The finite set of the *inference rules*, by whose virtue a formula is immediately derivable from a finite suitable set of formulae considered as premisses.

The notion of *formal system* corresponds to an improvement in the axiomatic method, J. Ladrière (19) tells us. J. Ladrière distinguishes four stages in the development of this method:

1. Intuitive axiomatics.
2. Abstract axiomatics.
3. Formal axiomatics.
4. The purely formal system.

The progress of axiomatics lies, according to Ladrière, in the gradual removal of intuition (20). «Any reference to a domain of sense outside the system is given up, states further the above quoted author, by using a symbolic, rigorously defined language».

We shall not specify the construction of formal systems. We will only add that the group of axioms should meet some conditions which may be generally summarized as follows: the axioms have to be *non-contradictory*, *sufficient* and *complete*.

Every formal system is built up in this way and this constructions means its *presentation*.

On the other hand, since a formal system is given, a determined sense can be assigned to the primitive components, by connecting them to some well defined objects. This correspondence is called the *representation* of the formal system and it thus achieves the «concretization» of the system (21).

Another correspondence species can be brought about: the connecting of elementary propositions with a certain class of statements whose truth (or falsity) is determined independently of the system. This particular correspondence is called the *interpretation* of the system.

«A formalized theory, write Fraenkel and Bar-Hillel (22), is usually set up in order to formalize some intuitively given theory. Whether, and to what degree, this aim is achieved, can only be determined after the formalized theory is provided with an *interpretation*, with the help of suitable *rules of interpretation*, turning thereby into an *interpreted calculus*. These rules can take many forms, but their common function is to provide each sentence of the formalized theory with a meaning, such that it turns into something that is either true or false, i.e. into a statement, though no effective method need of course be provided for deciding whether it is one or the other».

This amounts to the creation for the «formal system» of what is called a *model*. A model M is a set of elements put in correspondence with the components of a formal system S, so that:

- 1) to the propositions in the system S, there correspond statements made up with the elements of the set M;
- 2) irrespective of the system S, one can determine whether these statements are true or false;
- 3) true statements in M correspond to the propositions derived from S.

It therefore results that to interpret the formal system S means nothing else than to grant it a model M .

This operation, i.e. to put in correspondence a formal system S with a model M , is of semantic order (according to the usual terminology). The formal system S gives a *formal structure* to the set M , but – and we must strongly emphasize it – this correspondence does not *necessarily* proceed from the nature of the model M or the properties of the system S , but is freely chosen. Whence the result (natural under such conditions) that the same formal system S may allow several models (their amount being, in principle, unlimited). This state, an ineluctable consequence of the very idea of formal system, leads up to the admission of a relative and arbitrary formal structure of the model M under consideration. The organizing of the elements of the model M and of the true propositions in M is relative and arbitrary (to some extent), as well as the organizing of the system S .

In our conclusions we shall refer to this strange state which actually amounts to the following: the same formal system S may have a sequence (in principle unlimited) of various models M_1, M_2, M_3, \dots , and the same model M may be organized formally as a structure of a sequence (in principle unlimited) of various systems S_1, S_2, S_3, \dots

We thus actually admitted that there is no logico-formal structure of the model M (since it can be arbitrarily enough chosen). In other words, the relative and arbitrary construction of the formal system, the relative and arbitrary correspondence of the system S with its model M , assumes an axiom implicitly admitted, i.e. *there is no logico-formal intrinsic and specific structure of the model M* . At any rate, if there exists a specific structure of the model, it cannot be specifically described by a formal system just because it does not belong exclusively to the model. The true propositions in M are arbitrarily juxtaposed (to some extent), i.e. placed side by side as also primary etymological meaning of the concept of system – “????” – teaches us.

Remark.

We did not deal extensively with the details of the construction of a formal system, because we are interested in the *principle* of construction i.e. in an essential character of this principle, that will be further dwelt upon.

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RELATIVITY OF PRIMITIVE TERMS, AXIOMS AND THEOREMS IN FORMAL SYSTEMS

The same formal system may be built differently by starting from a certain group or primitive signs and axioms. In the following, we shall enlarge upon this idea that resulted in consequences beyond computation.

The idea that the primitive terms and the axioms can be freely chosen (on the principal condition of their non-contradiction) has been emphasized since the very beginning of the research on formalist axiomatics. Indeed, if «formalization» means the use of symbols deprived of any content, then it is obvious that there is no intrinsic sense of these symbols which could prescribe some of them prior to the others. Thus everything amounts to a game of symbols.

This has been perceived from the first attempts to formalize and axiomatize science. Here are Bertrand Russell's remarks on the logical formal system he (and Whitehead) presented in *Principia Mathematica* (23): «We have no reason to assume that it is impossible to find more simple ideas and axioms which could define and prove those we begin with. Everything it asserts is that ideas and axioms we begin with are sufficient and not that they should be necessary».

However, Russell, did not grasp the difficulty of the consequences resulting from the assumption of ideas and *axioms not necessary* in the construction of a system that wants to account for the *necessity* of conclusions within mathematics and generally within any other science.

Louis Couturat, who was as enthusiastic as Russell about the new logical-mathematical method, defined this conception even more boldly: «One should not give an absolute sense to the epithets of undefinable and unprovable, only related to a certain system of definitions and a certain order of proofs; in another system or another order, the same notions could be defined and the same propositions could be proved. Also there should not be assigned any absolute (epistemological) sense to the equivalent expressions of primitive notion and primitive proposition» (24).

The formalist logicians became gradually aware that the relativity of the primitive notions and of the axioms is directly and immediately resulting from the formalist conception itself, as such.

They thus accepted an actual state and concentrated it in one principle: that of the relativity of the choice of primitive symbols and of axioms. Since the formal systems are meant to replace the theory not only of any science but even of logics – thus becoming logical-formal systems – the above conclusions extended to every science in general. Here is what Rudolf Carnap writes: «According to the modern conception, this conclusions is no longer required as arbitrary propositions (*beliebige Sätze*) may be taken for axioms» (25),

Consequently, primitive notions, like axioms, are arbitrarily chosen. They do not impose by themselves at the beginning of a formal system, but are freely chosen, although, as a whole, they should satisfy some conditions.

«The passing to formal axiomatics, writes J. Ladrière, deeply alters the sense of axiomatics: the priority granted to some statements as starting point becomes actually wholly relative. It is no longer based on simplicity or a greater degree of evidence, but only on convenience. The choice of initial

statements entirely becomes arbitrary. In principle any system of statements may be taken as a system of axioms; the only thing we are interested in is the possibility to infer effectively the whole theory. In different words, the axiomatic system does not aim at enabling the appearance of the natural order existing among the statements of a theory, but at introducing an order that in itself may be any order whatever. The system is only asked to meet some simplicity and clarity conditions, and only these criteria will command the choice of axioms: the final goal is the removal of any ambiguity» (26).

It is apparent that the choice of primitive symbols and of axioms is exclusively «commanded» by a criterion of *practical* and not *theoretical* order (and in the given conditions they could not have been of another nature). The formalistic axiomatic position can be lastly summarized as follows: *we can stop wherever*. The construction of a system can be wherever started with.

The whole problem revolves around the initial choice of symbols and so-called axioms. This choice is arbitrary too.

«Generally, says Tarski, no fundamentally theoretical considerations decide upon the choice of a system determined by primitive terms and axioms among all equivalent systems: the reasons are rather of practical, educational and even esthetical order» (27).

However, such a conception deprives of any justification of «fundamentally theoretical order» the choice of the starting point and, as we shall further see, this total lack of logical initial foundation of a system bears on the whole system.

V.

FUNDAMENTAL OBJECTION

An objection whose decisive importance will be noticed by everybody, immediately opposes this essentially relative construction of any formal system of science.

Our objection regards every formal system which has been given an *interpretation*, i.e. a *model*. But even if this interpretation does not arrive to the construction properly speaking of a model, the formal system is touched nevertheless by our objection if inside its construction appear the notions (one or all):

- *definition*, with the sense of «it means»;
- *true* as opposite to *false* by means of *negation*;
- *demonstration*, with the sense that the demonstrative process leads to true formulae when starting from true formulae.

If the system remains to its level of purely algebraic game – as a Boolean algebra or even as a Lukasiewicz algebra – then we will make here no criticism to it. In a purely formal system, definition is only a conventional rule giving the possibility of replacement of a combination of signs by another such by proof. The idea of science and structure of combination; and the demonstration is only a transformation according to given rules of formulae into other formulae.

But as soon as one speaks of the «meaning» of a sign (or signs), the whole formal system is subjected to the objection that will be formulated below.

We shall formulate this objection in two parts. Since any set of well-chosen symbols may be taken as a set of primitive notions and any set of well-chosen formulas-theorems as axiomatic group, and the other notions and formulae could possibly be derived from the first ones, it ensues that:

- A. *The notions of a model organized by the whole formal system are reciprocally definable, i.e. they are definable one by the other, therefore by definitions, idem per idem, and consequently they are not in the least defined.*
- B. *Since the valid formulae of the system (axioms and theorema) are reciprocally provable the one by the other, it results that the proof of true propositions of a model organized by a system is circular and therefore they have no proof.*

Briefly, the axiomatic construction of a formal system gives no internal logical structure to a domain because, its internal «logical character» is circular and therefore vicious. All construction of such a system is *logically null*.

This objection of principle was discussed also by Aristotle in the *Second Analytics*.

As already mentioned, in his theory of science, Aristotle was necessarily confronted with two ways of knowledge: the immediate knowledge of principles and the mediate knowledge the science implies, according to Aristotle, «necessarily that provable science should start from immediate principles better known than the conclusion whose cause they are and that precede them» (28).

6 - The second opinion, according to which any knowledge is supposed to be the result of proof, is actually a second argument speculated upon by the sceptics. According to this conception, the problem of principles is no longer mooted and it is evinced that the proof is circular. This objection did not deny that all truths should be provable, but only that they are proved one by the other and therefore proofs are circular and reciprocal.

What answers the Master of Stageira? Here is his reply as stated in *Second analytics* (29): «Our theory is that it is not true that any knowledge is a proof; on the contrary, the knowledge of immediate premisses is independent of any proof. And this is obvious, since if we have to know the first premisses,

where every proof derives from and if the regress is to end in immediate truths, these truths have to be unprovable».

Ignoring, for a while, Aristotle's conception which shows that we possess the actual possibility to acquire the immediate knowledge (of principles), we shall bear in mind the following conclusions:

1. Principles must exist, we have to stop necessarily – αναγκη στηναι – otherwise we shall indulge *in regressus in infinitum*.
2. These principles are known and must be known otherwise than the conclusions, because, if we do not accept them, then there is only the possibility to accept one of the alternatives:
 - a) a dogmatic attitude;
 - b) relativistic and conventionalistic attitudes, both with no logical justification.

We shall further see the way Aristotle explains the logical possibility of the «immediate knowledge». For the time being, we shall only point out that the separation from *nature*, that he carries out between the initial principles of sciences and the theorems proved by them within science, is essential.

The above mentioned objection against the relative construction of any system, where neither «principles» nor «theorems» possess any logical foundation, must have been the goal of lengthy debates in Aristotle's time, because he himself feels compelled to consider and to reject it by his conception.

Indeed, within the same treatise, Aristotle ⁽³⁰⁾ deals with the objection against the demonstrative science.

1. Some claim, Aristotle says, that there are no principles and therefore proof is impossible, since then we should have to go down from proposition to proposition indefinitely, in a *regressus in infinitum*.
2. If, on the other hand, the series ends and there are principles (or prime premisses) these cannot be known, as there is no proof for them, which – for those raising this objection – was the only form of knowledge.

In short, since we are not able to know the prime premisses (or principles), the knowledge of conclusions deriving from them means no true knowledge and perhaps no knowledge at all, but only something based on the mere assumption that premisses should be true.

This argument against knowledge derived from principles has been speculated upon by the Sceptics who asserted, based on it, that «the beginning» should be dogmatic.

Against the circular proof, Aristotle says that it is no longer possible if we stop at principles which are better known and prior to conclusion, therefore the proof cannot start, in this case, from conclusions to prove principles.

If we do not accept these conclusions (irrespective of the assumed explanation) that principles are (and should be) simpler, better known and prior to conclusion, the respective scientific theory loses its logical character, i.e. any

external justification, and becomes an artificial construction, whose coherence is null, since it cannot say more than $A=A$, and where nothing is, strictly speaking, justified⁽³¹⁾.

It is remarkable, that the noetic nullity of all the systems (and of everything occurring within a system) has been entirely proved by Aristotle. Indeed Aristotle showed that those who do not admit the construction of theory in the above sense (according to his conception) admit the circular proof and reduce the theory «to the mere assertion that a thing exists because it exists, a frequent way to show anything»⁽³²⁾.

By assuming that the premiss may become conclusion and conversely, that the conclusion may become premiss, the circular proof does not justify either the premiss or the conclusion, and says essentially nothing else than that if something exists then this something exists, i.e. nothing.

In other words we have necessarily to stop, ἀναγκη στήναι, at knowledge acquired in another way than knowledge proved in a theory, since otherwise we necessarily stop, it is true, in order that the system could be expressed, but in a conventional and artificial way. But this cancels the logical character of the whole system, in the sense that there is no increase in knowledge due to this construction. (Pacius said: *ignota principia, ignotae conclusiones*).

The formal systems of science in general, and those of logic in particular, show that the argument of the great Stagirite has not been observed; hence the two contemporary positions: dogmatism and relativism (in its final stage, conventionalism).

Remark

A single contemporary logician seized the state, logic converted had been driven to. He is Ludwig Wittgenstein, at least in his *Tractatus Logico-Philosophicus*. Without referring to the *sine qua non* condition of a scientific theory as established by Aristotle, Wittgenstein surprisingly notices some important points, which confirm our analysis.

Indeed, here is what Wittgenstein writes on the «deduction» of logical tautologies from other tautologies: «The proof of logical propositions lies in the fact that we may create them starting from other logical propositions, by the successive application of operations which again engender some tautologies from the first ones (and tautologies result only from tautologies)»⁽³³⁾.

However, Wittgenstein had already noticed the wholly artificial character of such an extension, because he further adds: «Of course, such a way to show that the propositions are tautologies is not essential for logic. And because even propositions from which the proof starts *have to show, without proof, that they are tautologies*».

Here is how he defines more accurately the strange state of logic: «Within logic, the process and the result are equivalent»⁽³⁴⁾.

Therefore, since the proof does not bring anything new (*keine Ueberraschung*), it ensues that «*logic may be conceived, so that each proposition should be its own proof*. Every tautology shows by itself that it is a tautology» (Wittgenstein reaches the same conclusion as Aristotle: nothing justified – in the case under consideration – than by itself).

These remarks did not allow any longer to a logician like Wittgenstein to call the formal system of logic a «theory» (in the sense of science) and he is compelled to conclude: «Logic is no theory, but a reflexion of the world» (*die Logik ist keine Lehre sondern ein Spiegelbild der Welt*)⁽³⁵⁾.

We endeavoured to point out, by the aforesaid, that one of the most outstanding contemporary logicians denies that logic should be a «theory» and he denies this «theoretical» character of logic even to Frege's and Russell's logic (and implicitly to any «system of logic»), based on the argument that this «system» starts from a few propositions «which should show without proof that they are tautologies». Just because there is no «dignity» difference between axioms and theorems. In other words, the formal system of logic is no «theory» because it does not meet the requirement *sine qua non* of any theory stated by Aristotle.

VI.

LOGICAL CONVENTIONALISM

We already showed that if one admits that the axioms of a system can be arbitrarily chosen, there still exist but two possible positions:

1. the dogmatic position,
2. the relativistic position.

We shall but shortly mention the relativistic position, which decayed to an extremistic conventionalism.

Its theoretician is Rudolf Carnap, but this conception is accepted more or less openly by all who grow the symbolic logic into a formal system and generally by all who replaced scientific theories by formal systems, where the sign, empty of any content, is the fundamental element and the starting point (because we must necessarily stop somewhere, *αναμχλη στηναι*, as Aristotle said, in order to start), is arbitrarily chosen.

All logician mathematicians are actually conventionalists, because they admit the possibility to build equivalent logical systems starting from other primitive notions and axioms.

Even the intuitionists (Brouwer, Heyting, etc.) built their «formal» logic conventionally, by choosing some axioms with a view to reach anticipated results. It is but obvious that intuitionistic logic can be axiomatized starting

from other primitive ideas (other «factors») and from other axioms (and the same may be asserted also for polyvalent logics).

What drove Carnap to theorize conventionalism in logic, and generally in every science, was the example given by physicists upon whom the necessity was forced to admit various postulates arbitrarily, but able to save the coherence of their theories (as the postulate of contraction of moving bodies in the theory of relativity). The appearance of noneuclidean geometries and, eventually, Lukasiewicz's idea (1917) to build polyvalent logics, obviously induced one to think that logic itself – converted into a formal system – behaves similarly. The system built relatively and conventionally, replaced the theory of science and even logic.

In his work *Logische Syntax der Sprache*, Rudolf Carnap raises his logical conventionalistic conception to the rank of principle. According to Carnap, logic is a mere language and «everyone may construct his logic as it best suits him». There is no moral in logic – *in der Logik gibt es keine Moral* ⁽³⁶⁾.

The freedom to choose «one's logic», which becomes an entire freedom to choose conventions by virtue of which the logical system we can express within is built, was stated by Carnap as «the principle of tolerance» – *Toleranzprinzip*.

The only obligation left for he that freely chooses his «logic» or in Carnap's expression «a logical language» is to tell clearly whether he wishes to discuss with us, how he intends to proceed, i.e. how he is building syntactically his language ⁽³⁷⁾.

This extremistic standpoint was backed up, before Carnap, by K. Menger.

Menger thought, for instance, that the admission of the choice axiom (*Wahlaxiom*) in the set theory, may be regarded from the viewpoint of the history of science, either as one's dogmatic admission or as one's equally dogmatic rejection. However, the fact of admitting or rejecting this axiom is interesting from the standpoint of the mathematicians' biographers, says Menger, maybe even for history, but certainly not for mathematics and logic. The logician is concerned solely with the ensuing from the choice axiom ⁽³⁸⁾.

By reducing everything to formalism – the only way to be accurate and rigorous – Carnap is left with the naked formal expression of thinking, whose coherence – the only thing he still may ask, – is conventionally ordered. «Our task is not to establish prohibitions, he writes (all genuinely logical rules not freely chosen appear to him «as prohibitions») but to reach conventions». Carnap later thought that he might replace «the principle of tolerance» by «the principle of conventionalism» ⁽³⁹⁾.

It ensues from the above that those accepting the formal system of logic, built as already shown, do accept the *principia of conventionalism* in logic – and therefore in every formal system which assumes the formal system of logic (set theory, for instance) even if it does not fit accurately Carnap's view from the standpoint of the general conception of science. Yet, this concep-

tion is also rendered null by Aristotle's analysis; the scholastics summarized it by the formula *nulla est scientia*, there is no science, in this case.

We shall still mention, without pretending to deal with the history of the conventionalistic conception, that some «systeticians» admit a modified conventionalism and others even think that they get rid of conventionalism. N. Goodman, for instance, by denying the notion of «structure», asserts that we know the world as far as we describe it. For Goodman, to describe something means to express it in a schematical and conventional way ⁽⁴⁰⁾.

In other words, the knowledge of the world reduces to the construction of a descriptive «map» of the world, which summarizes the enormous amount of information about the world we can afford at a given moment, and which makes it intelligible.

However, Goodman clearly defines his conventionalism as not corresponding exactly to that of Carnap. Indeed, for Carnap, the choice of the logical language or even of a nominalistic or platonistic language is considered mere convention. In Goodman's conception not the choice of the logical language is a matter of convention; conventional is the way the reality is therein framed, i.e. the choice of the basical elements of the constructed system.

In order to escape the most critical problems implied by his conventionalist position, W.V. Quine adopts the analytical conception, which, according to him, should be apt to save a formal system and grant it significance. He asserts that every proposition of a scientific system is deprived of sense if taken alone. Only the totality of the propositions of a system, taken as a whole, may make sense ⁽⁴¹⁾; certain propositions have a central position, others a peripheral one within the system. The former should correspond, according to Carnap, approximately to synthetical propositions, the latter to the analytical propositions. Quine says that Kant's assertion that our knowledge is divided into separate synthetical and analytical propositions, with a sense as such, is a dogma deprived of any basis ⁽⁴²⁾.

We shall point out that Quine, like Goodman, reduced philosophy to the logistic analysis of the language.

It is however noteworthy that neither conception can get rid of the conventionalism of the constructed «logical» language, despite every effort to grant significance to systems deprived of any significance.

VII.

ARISTOTLE'S SOLUTION

We have already seen the conditions which, once satisfied, confer upon a corpus of ideas and propositions the quality of science. The principles should

be better known than the conclusion; they are anterior to, and simpler than, what is derived from them by way of proof.

The real construction of science implies two kinds of knowledge: 1) the knowledge of principles; 2) the knowledge of proofs, that, by the data of the problem, should be of a different nature as related to the knowledge of principles.

In other words, we have to stop somewhere – αναγκη στηναι – we have necessarily to stop at the axioms; but what forces us to accept some of them rather than the others?

It is noteworthy establishing that nobody could get rid of this absolute necessity, in the construction of a scientific theory, without ever stopping at axioms. However, on accepting them conventionally, the natural construction of a theory is faked and everythings reduced to the statement of vicious circles.

Let us now look at Aristotle's standpoint and the way he accepted axioms without proof. We shall not enter every detail of this conception, since we are only interested to show that the Stagirite put forward a solution enabling theory to operate, based on its structure, which he proved to be quite necessary.

Here is what he writes in the final chapter of Second Analytics⁽⁴³⁾: «We wish to clear now the notion of principle, i.e. how and by which faculty we come to know the principles. It has been primarily established⁽⁴⁴⁾ that we cannot possibly know anything by proof if we do not know the first immediate principles. As regards the knowledge of immediate principles, one can discuss whether it is of similar nature as the knowledge by proof, whether both kinds of knowledge are worth calling science or only one is a science and the other a kind of knowledge and finally, whether this faculty of knowledge – the principles – were born with us or unknowingly, whether they had not existed previously and were acquired».

Aristotle's conclusion is unambiguous: a special faculty of immediate knowledge of the principles as such, differing from knowledge by proof, must exist. Otherwise, science would not be possible. But science is possible and Aristotle already disposed of important and enough developed parts of the science: geometry, astronomy, arithmetics a.s.o. Hence, the faculty of knowledge, other than the proof, is implied by the existence of science itself.

Aristotle proceeds now to the explanation of the two sources of knowledge. To this end he distinguishes two parts of the human intellect, of the *Nous*: the passive intellect – νοϋζ παθητιχοϋζ – and the active intellect⁽⁴⁵⁾ νοϋζ πολητιχοϋζ.

The domain of science – επιστημη – based on experience, is the achievement of reasoning and of the passive intellect.

However, things do become intelligible but by the intervention of the active intellect. Induction, like proof, assumes certain principles and here we can see Aristotle's outstanding merit, since thus he connected science both to experience and intellect.

The basic principles of induction and proof are perceived by the active intellect through intellectual intuition, without any intermediary element. This intuition grants intelligibility to the principles of science, induction and proof. Without intellectual intuition, science wholly remains unintelligible, because neither induction nor proof can possibly explain themselves; instead they need principles and these are accounted for by the fact that they are perceived directly, by an act of intellectual intuition.

Briefly, Aristotle realized a theory by which he found the solution of the problem: how is science possible? This theory explains that the principles of science have a *sine qua non* warrant because they are simpler, prior to, and better known than the proves of science, namely by a direct act of the active intellect, which is the intellectual intuition.

It would appear as though Aristotle, trying to save science, was compelled to introduce an additional hypothesis, that of the active intellect, which assures the principles. This is not true. Indeed, the Stagirite asserts that the active intellect is the geometrical site of the intelligibles; in other words, perceiving the intelligibles, the active intellect merges to the intelligibles. Only in this he can possibly assert: *to know means to be*. The active intellect, by the act of intelligible perception, becomes the intelligible: the known object and the knowing subject blend at the summit of this noetic act and thus this becomes an ontological act. This was stated by Aristotle in his famous formulation: its thinking is thinking on thinking – η νοησιζ νοησεοζ νοησιζ. – This is the highest «dignity» ever granted to thought, since in Aristotle's conception, this dignity confers it a «divine» character.

We are obliged to acknowledge that, at least from the standpoint of the technique of logic, Aristotle's solution is out of the common. Indeed, the principles are certain, they possess an absolute certainty granted only by the active intellect, which, were it only a warrant for them, should constitute, from the logical standpoint, only an additional hypothesis.

Aristotle may therefore conclude: «Since only the active intellect is more truer than science, the principles are the object of the active intellect. This is true also because proof is not the principle of proof, hence neither science is the principle of science. If, besides science, we have no faculty to know the truth, the active intellect should be the principle of science as the totality of science is similarly related to the totality of things» (46).

In this way the principles of science are perfectly of stopping, because they are the active intellect itself, its existence within the act being but the principles. Every science starts from the active intellect and its explanation. Aristotle's formula – αναγγη στηναι – we have to stop – acquires a complete, precise meaning: we have to stop at the active intellect which merges into the intelligible essences and is their site. The act of stopping is an ontological one, since *to know* is identical, at this stage, with *to be*. We have thus necessarily to stop at principles because they are the principles, of the Being

and of the active intellect, which is, finally, in its noetic function, the Being itself (47).

Remark

We have not discussed the definition of truth in *Syntax* (Carnap) or in *Semantics* (Tarski), because these conceptions are already and implicitly touched by our previous analysis.

Alfred Tarski tried to construct a non-contradictory definition of truth in agreement with Aristotle's conception: «we should by our definition do justice to the intuitions which add here to the classical Aristotelien conception of truth» (48). To this purpose, Tarski demonstrates that the definition of truth needs a two-level language: a formal language (object-language) and a formal language which speaks about the first language and is its «metalinguage». The definition of the true proposition is possible only in the meta-language (which contains also the object-language).

The only thing we want to underline here is Tarski's incredible conclusion: «Consequently, Tarski writes, *we must always relate the notion of truth, like that of sentence, to a specific language: for it is obvious that the same expression which is a true sentence in one language can be false or meaningless in another*» (49).

Thus what is true can be false and what is false can be true!

It would be superfluous to discuss whether such a conception is able to give any noetic or ontologic foundation to the principles as such. But that was the question.

VIII.

CONCLUSIONS

Let us formulate now the conclusions called for by the preceding analysis.

1. Aristotle's reasoning is irrefragable. There is no science without principles – we have to stop at principles.
2. Any science whose principles are not assured otherwise than by the conclusions obtained by way of proof, is either dogmatic or conventionalistic, i.e. it is no science – *nulla scientia*, i.e. there is no real knowledge by such a construction.
3. Any scientific system conventionally, admitting «principles» brings about the relativity of principles and theorems and becomes circular. Nothing is any longer justified and the whole system results in an arbitrary, possibly completely coherent convention where, as already shown, one can say but A is A (herein lies the whole coherence of the system).

4. The essential problem of every scientific theory is therefore the finding of its principles, which, otherwise founded than the conclusions, will never be able to replace the latter within the theory ⁽⁵⁰⁾.
5. The domains organized by formal systems have no principles and hence no theorems, are no scientific theories, although, by their artificial construction, they take after the true scientific theory. They can tell, as Aristotle showed, but one thing: A is A (or, by introducing the negation: A cannot be but A; A cannot be non-A; A cannot be, and not to be, a.s.o.).
6. Particularly, the formalized logic, set forth by the contemporary logicians as a formal system, is not a science *-nulla scientia* ⁽⁵¹⁾.
7. The expression *nulla scientia* should be understood in accordance with the etymology of the term *scientia*: scientific systems are no epistemological theories with cognitive value even when they are given an «interpretation», i.e. when they are «concretized». Indeed, the only possibility a «system» has, is to «tie together», «to coordinate in a whole» by mere compatibility of the coordinated parts (i.e. by their non-contradictory juxtaposition). Since the coordination was achieved based on convention, it has nothing «theoretical», since knowledge cannot be the result of arbitrary conventions, admitted because they are convenient.

Remark

There is however a mathematical school, which up to a certain extent, estranges itself from the relativity of the formal systems. It is the intuitionistic school and especially the Dutch school of Brouwer and Heyting.

Contemporary intuitionism considers as its precursor Henri Poincaré, who was a fiery anti-formalist. «If the mathematical thought, he wrote, is reduced to void formulae, we are sure to cripple it». This, however, did not prevent him from conceiving science as «a wellshaped language», made up of «convenient» conventions. All the same, he granted intuition a place within the process of scientific knowledge (without accounting for this faculty as *contrepoint de la logique*). «Logic, he said, is not enough, intuition must preserve its rule as a complement ⁽⁵²⁾.

Here are the two main theses of the intuitionism, after Heyting ⁽⁵³⁾.

1. Mathematical has not an exclusively formal significance, but also a significance in the content.
2. Mathematical objects are immediately perceived by the thinking spirit; mathematical knowledge is independent of experience.

* * *

One might however object that certain formal systems, as those of logic, enjoyed resounding success by their practical applications in technics: electronic machines, automatic devices, translating machines, explanation of the neuron functions a.s.o., being thus brilliantly confirmed.

We agree that these achievements are beyond discussion; they are successful and valuable applications of the systems. But they represent no more than «practical applications». Starting with the Greeks and up to the moment it became a «system», science aimed at supplying us with knowledge about the reality. The replacement of science by conventionally constructed «systems» excludes scientific knowledge (*nulla scientia*), since the systems are deprived of any epistemological sense.

The application of an empty «system» or «algebra» to devices, translating machines a.s.o. proves the undeniable *utility* of formal constructions. Their entire value lies only in their utility.

Actually, two essentially different orientations are at stake. On the one hand Aristotle's science, governing up to the beginning of our century, with a clearly epistemological character; on the other hand, the present systems, directed toward technicality and ruled by the technicality of our times, which keep outside the problem of knowledge by their very nature (no one may know reality by conventions).

Aristotle's science proved to be a hierarchy of propositions, a hierarchy of truths and this means a pyramidal construction with base and peak, with hierarchical character both from the ontologic and noetic point of view (which, all things considered, coincide).

It is just this hierarchical character which lacks both in the formal system and the way in which it is interpreted. We could even say: the formal system is valid as long as it is a mere Boolean algebra (or, by extension, a Lukasiewicz algebra); as soon as we introduce the notions of *true* and *false* i.e. we interpret it, for instance, as logic, it is no longer anything – *nulla scientia*. In different words, the algebraic game of symbols is valid as such as long as we do not interpret it, or as long as its interpretation reduces to a mechanical model, i.e. a machine. In this case, the notions of true and false have not been introduced and the system does not acquire, by the mechanical interpretation, any epistemologic character. If we interpret, for instance, the two values of a Boolean algebra as «open» and «closed» (as in the circuits with contacts and relays), the formal system may be successfully applied. However, this application has nothing in common with the «true» and the «false».

It is out of question to diminish the part of any of these orientations to the benefit of the other. They are two different research directions, with their

own field. In this case, a «map» in Goodman's sense was drawn up, which expresses conventionally, the very general way every device runs, but this has nothing in common with epistemology.

From the standpoint of practice, nothing could be said against these systems; they are, on the contrary, technical successes. A design of the parts of a machine can be drawn, which is commonplace for engineers, but nobody could assert that this means «epistemological theory»; a map can also be drawn up, of the schematical construction and of the operation of automatic devices, but nobody could say that means «epistemological theory», except for those wishing to use metaphors. Goodman was right: in these cases, only a conventional map was drawn up, which should meet our practical requirements. As already shown, nothing more can be done under the given conditions, than to adopt a conventional position, i.e. whose only goal cannot and should not be but the practical goal.

This is clearly outlined also by the fact that between the way of development of a formal system and the way a machine operates there is identity in nature: the so-called proofs in the formal systems are no proper proofs, but only transformations of the initial data, as we have shown; but a machine, a device can do nothing more than transform, since this is their very mission and nature.

It is enough to consider any machine, those processing objects until those which transform energy, to become at one aware it. But «theory» does no reduce only to that, because in every theory the principles, in the Aristotelian sense, are better known, previously and immediately.

The conclusions which ensue cannot be regarded as a transformation of the axioms and do not reduce to them, because they represent something else than the principles they started from⁽⁵⁴⁾. Systems have, in the realm of abstraction, the same value as machines and devices in the realm of material applications. More precisely stated: «systems» are mechanisms or abstract machines. However, mechanisms or machines, all aim at taking man out of their operational circuit.

This is also the explicit goal of the interpreted «systems», to operate in themselves, without participation of the knowing subject, who constructed them⁽⁵⁵⁾. Therefore, a «system» (even when given a model) is not a science – *nulla scientia* – since knowledge is the specific act of the knowing subject, and in the case under consideration, it is a *sui-generis* act of the systems – or machine-creating subject. We could firmly assert that nothing of what it is created can provide knowledge independently of him who created it. The intelligible exists only in the presence of intelligence. And visualizing our conclusion in its broadest acception, i.e. in its universal content, we could say that *the intelligibility of creation lies in the effective presence of its Creator.*

NOTES

- 1 E. W. Beth, *The Foundations of Mathematics*, pp.31-32 (Second revised edition, North Holland Publishing Company, Amsterdam, 1965).
2. As far as it concerns Aristotle's axiomatics, see: H. Scholtz, *Die Axiomatik der Alten* (Blätter für deutsche Philosophie, vol. 4, 1930-1931).
3. Aristotle, *Second Analytics*, I, 33, 886.
4. F. Enriques, *Il concetto della logica dimostrativa secondo Aristotele* («Rivista di Filosofia», January, 1918).
5. J. M. Bochenski, *Elementa logicae graecae* (Roma, 1935).
6. *Second Analytics*, I, 31, 87b.
7. Op. cit., p. 32.
8. Op. cit., p. 33.
9. Op. cit., p. 34.
10. Op. cit., p. 36.
11. *Second Analytics*, I, 1, 71a.
12. *Ibidem*, 1, 2, 71b.
13. *Ibidem*, 1, 2, 71b.
14. We shall not enter the details of the «principles» which according to Aristotle, can be grouped as follows;
 1. axiom – αξιωμα;
 2. assertion θεσις;
 3. hypothesis – ιποθεσις;
 4. postulate – αιτημα;
 5. definition – ορισμοζ.
15. E. W. Beth, op. cit., p. 36.
16. Here is what Poincaré wrote in «*La Science et l'Hypothèse*», p. 66. Ed. Flammarion, Paris, 1909: «*The axioms of geometry are therefore neither synthetical a priori judgements, nor experimental facts. They are conventions; our choice, among all possible conventions, is guided by experimental facts; but it is free and limited only by the necessity to avoid any contradiction*».
17. David Hilbert, *Neubegründung der Mathematik*, (Abhandlungen math. Seminars, Hamburg, vol. I, 1922).
18. A. Frankel and J. Bar-Hillel, *Foundations of Set Theory*, p. 271, (North-Holland Publishing Co., Amsterdam, 1958).
19. J. Ladrière, *Les limitations internes des formalismes*, p. 35 (Gauthier-Villars, Paris et Louvain, 1975).
20. Op. cit., p. 36.
21. Op. cit., p. 42.
22. A. Fraenkel and J. Bar-Hillel, *Foundations of Set Theory*, p. 280.
23. A. N. Whitehead and B. Russell, *Principia Mathematica*, vol. I, p. VI (London, 1910).
24. L. Couturat, *Les principes des mathématiques*, p. 37 (Alcan, Paris, 1905).
25. R. Carnap, *Einführung in die symbolische Logik*, p. 172 (Wien, 1960).
26. J. Ladrière, op. cit., p. 37.
27. Alfred Tarski, *Sur la méthode déductive* (Travaux du IX-e Congrès International de Philosophie, vol. VI, p. 100, Hermann, Paris, 1937).
28. Op. cit., p. 1, 2, 71b.
29. Op. cit., I, 3, 72b.
30. Op. cit., I, 3, 72b.
31. The nullity of science, which makes no distinction of noetic nature whatever, between principles and conclusions, was emphasized, before Aristotle, by Plato himself. He accepted the same structure of the science as his pupil, because he writes: «Geometry and the science which accompany it (...) are longing to reach the Being and it will be impossible to see it with naked eyes as long as they (the geometricians) will make use of postulates and will support them without being able to prove their reason. *Indeed how could be called science a discipline ignoring its principle*

and whose conclusions and intermediate propositions rely on what it ignores» (Plato, *Republic*, 533 c).

This conception has been adopted by the logicians of the Middle Ages. Here is, for exemplification's sake, Julius Pacius' marginalia to the *Organon*, regarding the thesis of those who backed up the idea that there are no principles (in Aristotle's sense). In this case, if the principles are unknown (*ignota*), *necesse est etiam cetera, quae ex principiis demonstratur, esse ignota. Quare scientia nulla est.* («It is also necessary that the other propositions proved from principles, should be unknown. Therefore, there is no science»). See Julius Pacius, *Aristotelis Peripateticorum Principis Organum*, 2nd. ed., 1597, p. 419. Greek and Latin text with marginalia. Repro. reprint, Frankfurt am Main. 1967.

Indeed, the authors of such systems are compelled to accept circular definitions obviously. Here is how A. Fraenkel defines the cardinal number (*Abstract set theory*, p. 59, Amsterdam, 1961): «The cardinal number of a set *S* is the set of all sets equivalent with *S*». The author is well aware of the evanescence of such a proposition, because he adds that the definition appears somewhat paradoxical, but «such definitions are nowadays commonplace in mathematics».

The same definition is set forth by Russell under the form: «A number is what represents the number of a class» (*Introduction to Mathematical Philosophy*, Chapter II, London, 1919). Russell acknowledged that a definition appears like a vicious circle, but (like Fraenkel) he finds comfort in the idea that «such definitions are rather frequent». It is just Aristotle's conclusion: if principles are not better known than the conclusions and prior to them, then within the «system» one cannot say but $A=A$.

32. *Op. cit.*, I, 3, 73a. Aristotle shows here, with all necessary developments, that in such circular «theories» (nowadays accepted by mathematicians) everything amounts to: «if A exists, then A exists».
33. L. Wittgenstein, *Tractatus Logico-philosophicus*, prop. 6. 126 (ed. Kegan, London, 1933).
34. *Op. cit.*, prop. 6, 1261.
35. *Op. cit.*, prop. 6, 13.
36. R. Carnap, *Logische Syntax der Sprache*, p. 45 (Wien, 1934).
37. *Op. cit.*, p. 45.
38. K. Menger, *Der Intuitionismus (Blätter für deutsche Philosophie)*, 1930, p. 325).
39. R. Carnap, *Introduction to Semantics*, p. 247 (Cambridge, Mass, 1942).
40. N. Goodman, The way the world is (*The Review of Metaphysics*. 14, 1960).
41. M. V. Quine, *From a logical point of view* (Cambridge, Mass., 1953).
42. *Op. cit.*, p. 41.
43. II, 19.
44. *Op. cit.*, I, 2.
45. The name $\nu\omicron\zeta$ ποιητιχοζ actually appears with Aristotle's successors. He used to say $\nu\omicron\zeta$ $\alpha\pi\alpha\theta\eta\tau\iota\chi\omicron\zeta$, the non-passive intellect. Here is how Aristotle explained the necessity of the two «parts» or the intellect: (*De Anima*, III, 5, 430a) «Since there exists in nature, on the one hand the matter in potentiality, i.e. all individuals of the same genus and, on the other hand, the efficient cause, because it gives to each individual its form, since this shapes the matter; these two things must necessarily exist in the soul. We must therefore distinguish between two intellects: the former representing the matter of thought, the latter its cause and form».
46. *Second Analytics*. II, 19, 100a.
47. The stopping act must be related, for a complete understanding, to the prime mover – $\pi\rho\omega\tau\omicron\nu$ $\chi\iota\nu\omicron\upsilon\nu$ – which, according to Aristotle, although it operates everything, must be necessarily motionless. Thus, the formation of thought must originate in a non-formation, stopping act, as repeatedly emphasized by Aristotle, for instance, in *Physics*, VII, 3, 247b: «To acquire science primarily does not mean, birth because we say to know and to think means that reason is at rest and stopped, and there is no birth at rest».

48. See A. Tarski: *Der Wahrheitsbegriff in den formalisierten Sprachen* («Studia Logica», Leopoli, 1935): *The semantic conception of truth* («Philosophical and phenomenological Researches», 1944).
49. *The semantic conception of truth*, p. 14.
50. Along the same line, it is worth mentioning J. L. Destouches' works, who analyzed in an interesting and original manner the physical-mathematical theories. Looking into the preliminary part of the physical theory. Destouches writes: «Thus, every physical theory is not merely a deductive theory: if it were to have this character, it should first of all include a non-deductive part preceding the axiomatic statement, and this part was called by me inductive synthesis. Destouches finds three parts in a deductive theory: 1) *inductive synthesis*; 2) *axiomatic statement*; 3) *proper deductive theory*. According to Destouches, the inductive synthesis does not reduce to induction, it is larger than writes, is either explanation or suggestion, by any possible means, of all elements leading to the introduction of as certain notion or postulate». We consider the introduction of the notion «inductive synthesis» Aristotle's necessity to stop at notions and principles otherwise known than the conclusions. See J. L. Destouches: *La synthèse inductive comme fondement des concepts et des énoncés primitifs d'une théorie physique* (in *The foundations of statements and decisions*, edited by A. Ajdukiewicz, p. 240, Warsaw, 1965). The idea already appears in Destouches' thesis: *Essai sur la forme générale des théories physiques* (Cluj, Roumanie, 1938). It is also explained by the author in his *Principes fondamentaux des physiques théoriques* (3 vol. Paris, 1942). We should still add that every science drawing its principles exclusively upon experience wholly meets the conditions established by Aristotle for any science, since each system of axioms of the science is *otherwise* known than the conclusions. But the respective science loosens its epistemological ability, the very moment arbitrary principles, axioms, postulates etc. which have no other justification but to make science «convenient» and thus be «useful» and hence, by their nature, liable to be replaced, enter its corpus.
51. We have to draw attention to the fact that Aristotle did not consider logic a theory. The scholastic logicians, heirs of the Aristotelian capital, maintained all along Middle ages, that logic is no science, but *modus scientiarum*, the mode, the principle, the procedure of the other sciences. See, for further developments, our study *La Logique classique et les systèmes formels* («Revue Roumaine des Sciences Sociales», série de Phil et Logique, 2, 1966).
52. H. Poincaré, *La valeur de la Science*, Paris, 1905.
53. A. Heiting, *Mathematische Grundlagenforschung, Intuitionismus. Beweistheorie* (p. 3, Springer, Berlin, 1934).
54. We shall emphasize the connection Prof. Edward G.B. Ballard makes between a system of conventions, which he considers a «*thought-model*», and the construction of a machine. At the end of Renaissance, he says, and at the beginning of modern times, the machine became the king of the thought-model. World was considered a machine, and the conventions, by the terms of which the work of a machine was explained, were used to explain the work of universe». Without doubt, Laplace's idea, who viewed an *Intelligence* knowing the positions and velocities of every particle in the universe and hence the past and future of the universe is also due to the *thought-model*, automatic device constructed analogously to a machine, by conventions. (E. G. Ballard: *Reason and Convention*, «Tulane Studies in Philosophy» vol. I, p. 21-42, New Orleans, 1964).
55. We must underline here an interesting idea of the Italian thinker, Prof. Aldo Testa, author of the philosophical conception «Dialogica». He makes a neat separation between the «structure system» and «observation system» (*sistema di struttura* and *sistema di osservazione*). In his opinion, the consideration of the «system» before being observed, that is without including the observer himself, is a fundamental error, because the observer himself is one of the elements of the system (See, Aldo Testa: *Determinismo ed Indeterminismo: Discorso di Fisica; La Relatività Universale*, etc., «Biblioteca di Cultura Filosofica» – Cappelli, Bologna).